Relational semantics: from simple to non-idempotent intersection types and back

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Relational semantics is a simple and well-studied denotational model for the untyped λ-calculus [5, 14, 3]. Inspired by Girard's linear logic [10, 11, 4], it can be presented syntactically as a non-idempotent intersection type system [9, 13, 12] and provides qualitative and quantitative information, such as the characterization of normalizing terms and their execution time (i.e., the number of steps to reach the normal form) [7, 8, 6, 2, 1].

We study the relational semantics for the simply typed λ-calculus extended with the fixpoint combinator; we show that the interpretation of a simply typed term is nothing but the interpretation of the untyped term restricted to the non-idempotent intersection types refining the simple type. Our approach is based on merging a simply typed derivation with a non-idempotent intersection one, which is not trivial since simply typed terms may not enjoy subject expansion (because simple types are too restrictive) or be normalizing (because of the fixpoint combinator). We also show that some (but not all) well-known qualitative and quantitative information provided by relational semantics and non-idempotent intersection type derivations for the untyped λ-calculus lifts to the simply typed λ-calculus with fixpoint combinator almost for free, thanks to our results.

Our contribution is distinct from Pautasso' and Ronchi Della Rocca's [15] in several respects:

1. Pautasso and Ronchi Della Rocca use a variant of non-idempotent intersection types with the same expressive power as simple types (hence they can type less terms than us).

2. The non-idempotent intersection type system used by Pautasso and Ronchi Della Rocca provides different qualitative and quantitative information than the ones we use.

3. Pautasso and Ronchi Della Rocca use a different technique and their results rely on the fact that simple types guarantee strong normalization, hence their approach cannot be extended to the simply typed λ-calculus with fixpoint combinator.

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